Fast Enumeration of Smallest Engineering Strategies in Genome-Scale Metabolic Networks

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Outline



1) Introduction: Stoichiometric and Constraint-Based Analysis of Biochemical Reaction Networks

→ Focus: Elementary Modes and Minimal Cut Sets

2) (Re)Designing Metabolic Networks by Minimal Cut Sets

3) Duality Between Modes and Cuts and Algorithms for Their Computation

Methods will be presented with focus on metabolic network analysis.

In principle, they are applicable to any network of chemical reactions or material flows.

Metabolic Reaction Networks





Metabolic Network Modeling







Meanwhile, >100 genome-scale reconstructions of organism-specific metabolic networks have been published.

They typically contain 600 - 3000 or more reactions and metabolites.

For certain aspects, it is often sufficient to focus on the core of the central metabolism with 100-150 reactions.

Stoichiometric Matrix N



 $m \ge q$ stoichiometric matrix **N** : captures the stoichiometric coefficients

- *m* (internal) metabolites (external sinks, sources, ... often not considered in **N**)
- q reactions (may include pseudo reactions, e.g., transport or biomass synthesis)
- *Rev*: indices of reversible reactions; *Irrev* = indices of irreversible reactions



(store additonally the enzymes/genes for each reaction)

Graphs vs. Hypergraphs



Network = Graph?!

(Metabolic) Reaction networks span a hypergraph and N is the incidence matrix of this hypergraph.

Graph: $A \rightarrow B$ **Edges** can only connect pairs of nodes.

A (bimolecular) reaction $A + B \rightarrow C + D$ cannot be represented in a graph.

Hypergraph: Hyperedges may connect two sets of nodes.



hyperedge $h = \{ \{A, B\}, \{C, D\} \}$

Hypergraphs can be converted to graphs where some important network properties can be studied. However, graph representations are limited for *functional* network analysis.

More on hypergraphs in biological network analysis: Klamt S, Haus U, Theis F: Hypergraphs and Cellular Networks. 2009. PLoS Computational Biology 5:e1000385.

Stoichiometric Matrix N



N is fundamental for dynamic modeling of (metabolic) reaction networks (ODEs):

$$\frac{d\mathbf{c}(t)}{dt} = \mathbf{N} \cdot \mathbf{r}(t)$$

c(t) : metabolite concentrations at time t [mmol/gDW]r(t) : (net) reaction rates [mmol/(gDW \cdot h)]

 $\mathbf{r}(t) = f(\mathbf{c}(t), \mathbf{p}) \longrightarrow$ detailed knowledge of reaction mechanism, kinetic parameters and regulation required (often not available)

Methods for stoichiometric / structural / topological network analysis rely on N and do / need not consider reaction kinetics.

Quasi-Steady-State Assumption



- high turnover of metabolites in central metabolism (typical turnover time: 0.1-60 sec)
- reasonable assumption: metabolism operates close to steady-state, in particular in micro-organisms (e.g., bacteria, yeast,...)

$$\frac{d\mathbf{c}(t)}{dt} = \mathbf{N} \cdot \mathbf{r}(t) \approx \mathbf{0}$$

$\mathbf{N} \cdot \mathbf{r} = 0$	Metabolite Balancing Equation					
	(Production = Consumption					
	for each <u>internal</u> metabolite)					

$$\mathbf{r}_{Irrev} \ge \mathbf{0}$$
 Irreversibility Constraints

 \rightarrow equalities and inequalities (constraints)

 \rightarrow "constraint-based modeling of metabolic networks"

Space of feasible steady state flux distributions



C1) *Quasi-steady-state*: **Nr** = **0**

Constraint-based Methods:

- Metabolic Flux Analysis
 (particular flux distribution)
- Flux Balance Analysis (optimal flux distribution)
- Elementary Modes
- Minimal Cut Sets

Solution space of flux vectors r: null space (or kernel) of N: ker(N)

Convex polyhedral cone (flux cone)



Elementary (Flux) Modes



(Schuster et al.)

An Elementary (flux) mode (EM): is a flux vector e fulfilling

- C1) Steady State: Ne = 0
- C2) *Irreversibility*: $\mathbf{e}_{Irrev} \ge \mathbf{0}$
- C3) *Elementarity*: no vector $\mathbf{v}\neq\mathbf{0}$ satisfies C1 and C2 and supp(\mathbf{v}) \subset supp(\mathbf{e}) where supp(\mathbf{v})={i | $v_i \neq 0$ }

Elementary modes:

- support-minimal sets of reactions able to perform function in steady state
- correspond to pathways or cycles
- uniquely identified by its participating reactions (rate $\neq 0$)



• deletion of a reaction: new set of EMs can easily be derived.

Elementary (Flux) Modes



• elementary modes generate the flux cone (by non-negative linear combinations)

$$\mathbf{r} = \sum \gamma_i \mathbf{e}^i \quad ; \quad \gamma_i \ge 0$$

- each extreme ray of the flux cone is an EM (not all EMs are extreme rays)
- computation of EMs relies on methods for computing extreme rays of polyhedral cones
- combinatorial complexity: even in medium-scale networks millions of EMs may exist
 → applicable to medium-scale networks only (<150 reactions)



Elementary Modes: A Versatile Tool for Metabolic Pathway and Network Analysis



Biochemical Network

Functionality of interest

Minimal Functional Units (Elementary Modes)

- pathways and routes
- flexibility to perform a task
- optimal performance
- functional importance of reactions
- phenotype predictions
- structural couplings



Central metabolism of *E.coli* : **Elementary-modes analysis**

6-Phospho

21592 (80%)

Glucose

27100

NADPH



all 4 substrates

simultaneously

507633

- - \times



Pyr ###

PEP

Glucose

File Edit Tools Window Help FluxAnalyzer

number of EMs

with biomass synthesis

thereof:

110 reactions89 metabolites

(Central metabolism in detail; anabolic part compressed)



Acetate

363 (61%)

598

Glycerol

9479 (84%)

11333

Succinate

3421(80%)

4250

Substrates:

Glucose Succinate Glycerol Acetate

Stelling, Klamt et al. ; Nature 420:190-193



Product yields and optimal EMs



Optimal yield can be identified AND all the modes by which the optimum is achieved



Example: biomass yield of E.coli on (substrate) succinate

Metabolic Engineering / Biotechnology



Metabolic Engineering: experimental + theoretical techniques for the targeted improvement of metabolic pathways in microorganisms for the production of fuels and chemicals from renewable resources.



Metrics: Concentration (g/L), Yield (g/g), Rate (g/L/h and g/gDW/h)

Large-scale production process

Computational Strain Design (Metabolic Engineering / Biotechnology)





Metabolic Engineering: (Re-)Design of Cellular Factories





Constraint-Based Methods for Computational Strain Design



Bi-level optimization problems

- knockout strategies enforcing coupled biomass and product synthesis
- OptKnock and variants OptReg, OptForce (Maranas et al.); OptORF (Reed at al.); EMILiO (Mahadevan et al.); and others
- \rightarrow MILP problems solved at genome-scale
- \rightarrow difficult: #knockouts >10 ; #potential targets >300;
- \rightarrow #alternative strategies found <100

Elementary-modes-based optimization

- Minimal metabolic functionality (Trinh, Srienc et al.); FluxDesign (Wittmann et al.); CASOP (Hädicke et al.); and others
- \rightarrow not applicable to genome-scale networks

(Constrained) Minimal Cut Sets: systematic enumeration of all minimal knockout sets that block *undesired* while keeping *desired* metabolic functions.

- enumerative approach
- calculation based on elementary modes
 - \rightarrow <u>so far</u>: not applicable to genome-scale networks

From Elementary Modes to Minimal Cut Sets



Elementary Modes: minimal functional units (pathways) that can operate in steady state



Minimal Cut Set (MCS): minimal set of cuts (knockouts) blocking certain functions in steady state

MCSs are the minimal hitting sets of the corresponding EMs

Max-Planck-Institut Magdeburg Klamt S: BioSystems 2006, 83:233-237.

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From Elementary Modes to Minimal Cut Sets



Elementary Modes (EM): minimal functional units that can operate in steady state



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Elementary Modes, Minimal Cut Sets and Minimal Hitting Sets



Computation of Minimal Hitting Sets (here: for a given set of Target Modes):

Many algorithms are known from hypergraph theory.

Algorithm of Claude Berge [1] performs well for metabolic networks [2].

[1] Claude Berge. (1989): Hypergraphs. Combinatorics of finite sets.(North-Holland, Amsterdam) [2] Haus U, Klamt S, Stephen T. (2008) Computing knockout strategies in metabolic networks. J Comp Biol 15: 259-268.

Elementary Modes, Minimal Cut Sets and Minimal Hitting Sets





Generalization: Constrained Minimal Cut Sets



Elementary Modes: minimal functional units (pathways) that can operate in steady state





Constrained Minimal Cut Set (MCS): minimal cut sets <u>blocking undesired</u> while <u>keeping desired</u> metabolic behaviors.

Constrained MCSs: A Tool for Computer-aided Redesign of Metabolic Networks



Constrained Minimal Cut Set (cMCSs) problem for reaction networks:

- set *T* of target EMs (to be blocked)
- \bullet set ${\mathcal D} \, \text{of desired EMs}$
- *n*: minimal number of desired EMs in \mathcal{D} that must not be hit by the MCSs
- algorithm for minimal hitting set calculation adapted for cMCSs:
 - a) check on-the-fly whether MCS candidates keep desired modes (often faster)
 - b) identify constrained MCSs from unconstrained MCSs during post-processing
- large variety of complex intervention problems can be conveniently formulated and solved by cMCSs
- exhaustive enumeration of all knockout strategies
 - (... if the network is not too large)

Using Constrained Minimal Cut Sets for Strain Design



(1) Compute and analyze space of feasible conversions and specify desired and undesired behaviors

(3) Example of the remaining solution space for one specific constrained MCS



Example: Coupled Biomass and Ethanol Synthesis in *E.coli*



Goal: search for interventions that lead to high (anaerobic) ethanol synthesis on glucose while still enabling some formation of biomass



Max-Planck-Institut Magdeburg

Hädicke and Klamt (2011): Computing Complex Metabolic Intervention Stratgeies using Constrained Minimal Cut Sets. Metabolic Engineering, 13:204-213.

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Duality: Elementary Modes and Minimal Cut Sets



EMs and MCSs are dual representations of network functions.



MCSs are Minimal Hitting Sets of EMs



Elementary Modes: minimal functional units (pathways) that can operate in steady state



MCSs are Minimal Hitting Sets of EMs



Elementary Modes: minimal functional units (pathways) that can operate in steady state



MCSs are Minimal Hitting Sets of EMs



Elementary Modes: minimal functional units (pathways) that can operate in steady state



EMs are Minimal Hitting Sets of MCSs!



Elementary Modes: minimal functional units (pathways) that can operate in steady state



EMs are Minimal Hitting Sets of MCSs!



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Ballerstein K, von KampA, Klamt S and Haus UU. (2012) Minimal cut sets in a metabolic network are elementary modes in a dual network. Bioinformatics 18: 381-387.

Elementary Modes and Minimal Cut Sets: Algorithms and Duality





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Computation of MCSs



Unfortunately:

- computation of MCS as EMs in the dual does not pay off in most cases!

Full enumeration of MCSs/EMs anyway not realistic in genome-scale networks!

Focus on smallest MCSs reasonable for applications! → But: how to compute?



→ Enumerate smallest MCSs as shortest EMs in the dual network!

Elementary Modes and Minimal Cut Sets: Algorithms and Duality





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Defining Undesired und Desired Metabolic Functions MAX-PLANCK-GESELLSCHAFT *R*2 *R1* **S** ⁻ Production of P1 undesired! **R5** Production of P2 desired! **Undesired (targeted) function** Specification ... **Desired function** Target / Undesired modes Desired modes ... via elementary flux modes

... via equalities and inequalities on fluxes $r_{4} > 0 \qquad \qquad r_{5} > 0 \\ (r_{4} = 0) \\ \hline \mathbf{T} \mathbf{r} \leq \mathbf{b} \qquad \qquad \mathbf{D} \mathbf{r} \leq \mathbf{a}$

MCSEnumerator: Computation of Smallest (Constrained) MCSs in Genome-Scale Networks





von KampA and Klamt S (2014): Enumeration of smallest intervention in genome-scale metabolic networks. PLOS Comp Biol, 10: e1003378. <u>MILP problem for shortest EMs</u> Integer variables: $z_i=0 \leftrightarrow v_i=0$; $z_i=1 \leftrightarrow v_i\neq 0$ Objective function: *minimize* $\sum z_i$

MCSEnumerator: Computation of Smallest (Constrained) MCSs in Genome-Scale Networks



4.) Enumerate shortest EMs in the dual by solving several MILP problems.
 Normally: iteration k delivers k-th shortest EMs.
 MCSEnumerator: iteration k delivers all EMs of size k.

- 5.) The found **shortest EMs in the dual** correspond to the **smallest MCSs** for the (primal) undesired behaviors.
- 6.) Filter *constrained* MCSs maintaining desired behavior
 - \rightarrow Solving one LP per MCS (low computational demand).





Application Examples

of constrained MCSs in large-scale metabolic networks

Example 1:

Enumeration of Synthetic Reaction Lethals in E.coli model iAF1260



Synthetic reaction lethals (SRLs) = MCSs that block growth

```
Undesired behavior: \mu \ge \mu_{min}
Desired behavior: { }
Model: iAF1260 (2077 reactions; 1387 metabolites)
```

Original study: Suthers et al. (2009, Mol Syst Biol)

→ SL Finder (bi-level optimization): full enumeration of SRLs up to 3 knockouts

MCS size	number of MCSs	runtime* SL Finder	runtime* <i>MCSEnumerator</i>
1	277	11.1 s	11.1 s
2	96	1.5 h	39.1 s
3	247	74.1 h	16.8 min
4	402		18.5 h
5	1464		410.4 h

* Computed on mini-cluster with 2 Intel Xeon DP X5650 (6 cores each) (runtime on typical quadcore is ~1.5 higher)

→ all SRLs up to size 5 can now be computed → >100 times faster than SL finder

Example 2: constrained MCSs for coupling anaerobic growth and ethanol synthesis in *E.coli* (model: iAF1260)





von KampA and Klamt S (2014): Enumeration of smallest intervention in genome-scale metabolic networks. PLOS Comp Biol, 10: e1003378.

Example 2: constrained MCSs for coupling anaerobic growth and ethanol synthesis in *E.coli* (model: iAF1260)



Scenario	$Y_{Eth/Glc}^{\min}$	# MCSs	# cMCSs		cMCSs size					runtime	
				1	2	3	4	5	6	7	[h]
1	1.4	156477	8819	0	0	2	98	533	1737	6449	16.6
2	1.8	138675	4618	0	0	2	70	509	917	3120	20.9

Some results / conclusions:

- \rightarrow Full enumeration of cMCSs up to size 7 within one day.
- → More than 4000/8000 intervention strategies found. Ony 3-6% of the computed MCSs (>100,000) are valid cMCSs.
- \rightarrow Many cMCSs correspond to well-known strategies.
- → Some cMCSs with only 3 knockouts; even for high-yield strategy.
 → Experimental validation ongoing.
- \rightarrow Pick optimal intervention strategy via performance measures ...

Picking Optimal Knockout Strategies via Performance Parameters



Min-Ethanol-Yield vs. Max-Growth-Rate for all cMCSs of scenario 1 ($Y_{Eth/Glc}^{min} = 1.4$)



von KampA and Klamt S (2014): Enumeration of smallest intervention in genome-scale metabolic networks. PLOS Comp Biol, 10: e1003378.

Example 3: constrained MCSs for coupling photosynthetic growth an ethanol synthesis in cyanobacteria





Key results:

- joint work with Steuer group (HU Berlin)
- genome-scale model of *Synechocystis* sp. PCC 6803 (Knoop et al. (2013))
- undesired behavior:
 - \rightarrow low yield of ethanol (per mmol photons)
- desired behavior:
 - \rightarrow high yield of ethanol and growth possible
- **coupling** of biomass and ethanol synthesis under photoautotropic conditions **much harder** than in heterotrophic organisms: \geq 14 knockouts required
- all cMCSs of size 14, 15 and 16 enumerated within one day (1564 cMCSs)
 - \rightarrow reveal principles of suitable coupling strategies
 - → selected strategies under experimental validation (with Algenol Biofuels Germany)

Recent Work: Extension to Regulatory cMCSs



- joint work with Radhakrishnan Mahadevan (Toronto)
- extend concept of cMCSs to allow also up / downregulation of metabolic fluxes
- idea: represent up / downregulation as "cuts":

Representing upregulation as a reaction cut





Knockout of r_{S} induces upregulation of flux $r_{A_B} (\geq \mathbf{K})$



Knockout of r_S induces downregulation of flux $r_A = (\leq \mathbf{K})$

Mahadevan R, von Kamp A and Klamt S: Geneome-scale strain design based on regulatory minimal cut sets. Submitted

CellNetAnalyzer





MATLAB toolbox for Biological Network Analysis.

Large number of methods for stoichiometric and metabolic network analysis including computation and analysis of EMs and (c)MCSs).

Free for academic use: http://www.mpi-magdeburg.mpg.de/ projects/cna/cna.html



Conclusions



- Stoichiometric Network Analysis: an established framework for exploring (large-scale) metabolic networks.
- Elementary Modes: for studying functional properties of metabolic networks.
- **Constrained Minimal Cut Sets**: a universal and flexible approach for formulating and solving intervention problems in reaction networks.
- EMs and MCSs are **dual representations** of network functions.
- **Duality** between EMs and MCSs offers new <u>computational</u> and <u>conceptual</u> perspectives for exploring functions and design strategies in metabolic networks.
- New and effective algorithm for enumerating smallest (c)MCSs in genome-scale networks (via shortest EMs in the dual system)



Related topics in ecological/ecosystems modeling:

- stoichiometric models of microbial communities (work in progress in our lab)
- material flow analysis in ecosystems (e.g., in food webs, biogeochemical models)
- "ecological stoichiometry"

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